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Provably accurate simulation of gauge theories and bosonic systems

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Abstract

Quantum many-body systems involving bosonic modes or gauge fields have infinite-dimensional local Hilbert spaces which must be truncated to perform simulations of real-time dynamics on classical or quantum computers. To analyze the truncation error, we develop methods for bounding the rate of growth of local quantum numbers such as the occupation number of a mode at a lattice site, or the electric field at a lattice link. Our approach applies to various models of bosons interacting with spins or fermions, and also to both abelian and non-abelian gauge theories. We show that if states in these models are truncated by imposing an upper limit Λ on each local quantum number, and if the initial state has low local quantum numbers, then an error at most ε can be achieved by choosing Λ to scale polylogarithmically with $1/\varepsilon$, an exponential improvement over previous bounds based on energy conservation. For the Hubbard-Holstein model, we numerically compute a bound on Λ that achieves accuracy ε, obtaining significantly improved estimates in various parameter regimes. We also establish a criterion for truncating the Hamiltonian with a provable guarantee on the accuracy of time evolution. Building on that result, we formulate quantum algorithms for dynamical simulation of lattice gauge theories and of models with bosonic modes; the gate complexity depends

almost linearly on spacetime volume in the former case, and almost quadratically on time in the latter case. We establish a lower bound showing that there are systems involving bosons for which this quadratic scaling with time cannot be improved. By applying our result on the truncation error in time evolution, we also prove that spectrally isolated energy eigenstates can be approximated with accuracy ε by truncating local quantum numbers at Λ=polylog(1/ε).