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Clustering of conditional mutual information and quantum Markov structure at arbitrary temperatures

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Abstract

Recent investigations have unveiled exotic quantum phases that elude characterization by simple bipartite correlation functions. In these phases, long-range entanglement arising from tripartite correlations plays a central role. Consequently, the study of multipartite correlations has become a focal point in modern physics. In these, Conditional Mutual Information (CMI) is one of the most well-established information-theoretic measures, adept at encapsulating the essence of various exotic phases, including topologically ordered ones. Within the realm of quantum many-body physics, it has been a long-sought goal to establish a quantum analog to the Hammersley-Clifford theorem that bridges the two concepts of the Gibbs state and the Markov network. This theorem posits that the correlation length of CMI remains short-range across all thermal equilibrium quantum phases. In this work, we demonstrate that CMI exhibits exponential decay concerning distance, with its correlation length increasing polynomially in relation to the inverse temperature. While this clustering theorem has previously been established for high temperatures devoid of thermal phase transitions, it has remained elusive at low temperatures, where genuine long-range entanglement is corroborated to exist by the quantum topological order. Our findings unveil that, even at low temperatures, a broad class of tripartite entanglement cannot manifest in the long-range regime. To achieve the proof, we establish a comprehensive formalism for analyzing the locality of effective Hamiltonians on subsystems, commonly known as the 'entanglement Hamiltonian' or 'Hamiltonian of mean force.' As one outcome of our analyses, we enhance the prior clustering theorem concerning

bipartite entanglement. In essence, this means that we investigate genuine bipartite entanglement that extends beyond the limitations of the Positive Partial Transpose (PPT) class.